Implicit Differentiation

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Questions in past papers often come up combined with other topics.
Topic tags have been given for each question to enable you to know if you can do the question or whether you need to wait to cover the additional topic(s).

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Question 1

Qualification: AP Calculus AB

Areas: Differentiation, Applications of Differentiation

Subtopics: Implicit Differentiation, Tangents To Curves, Vertical Tangents

Paper: Part B-Non-Calc / Series: 2000 / Difficulty: Hard / Question Number: 5

5. Consider the curve given by $xy^2 - x^3y = 6$.

(a) Show that $\dfrac{dy}{dx} = \dfrac{3x^2y - y^2}{2xy - x^3}$.

(b) Find all points on the curve whose x-coordinate is 1, and write an equation for the tangent line at each of these points.

(c) Find the x-coordinate of each point on the curve where the tangent line is vertical.

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Question 2

Qualification: AP Calculus AB

Areas: Differentiation, Differential Equations

Subtopics: Implicit Differentiation, Separation of Variables in Differential Equation, Particular Solution of Differential Equation, Integration Technique – Standard Functions

Paper: Part B-Non-Calc / Series: 2001 / Difficulty: Somewhat Challenging / Question Number: 6

6. The function f is differentiable for all real numbers. The point $\left(3, \frac{1}{4}\right)$ is on the graph of $y = f(x)$, and the slope at each point (x, y) on the graph is given by $\frac{dy}{dx} = y^2(6 - 2x)$.

(a) Find $\frac{d^2y}{dx^2}$ and evaluate it at the point $\left(3, \frac{1}{4}\right)$.

(b) Find $y = f(x)$ by solving the differential equation $\frac{dy}{dx} = y^2(6 - 2x)$ with the initial condition $f(3) = \frac{1}{4}$.

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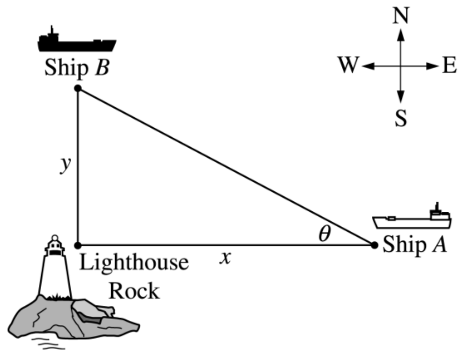
Question 3

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Differentiation

Subtopics: Rates of Change (Instantaneous), Related Rates, Implicit Differentiation, Modelling Situations, Differentiation Technique – Standard Functions, Differentiation Technique – Trigonometry, Differentiation Technique - Quotient Rule

Paper: Part B-Non-Calc / Series: 2002-Form-B / Difficulty: Hard / Question Number: 6



6. Ship A is traveling due west toward Lighthouse Rock at a speed of 15 kilometers per hour (km/hr). Ship B is traveling due north away from Lighthouse Rock at a speed of 10 km/hr. Let x be the distance between Ship A and Lighthouse Rock at time t, and let y be the distance between Ship B and Lighthouse Rock at time t, as shown in the figure above.

(a) Find the distance, in kilometers, between Ship A and Ship B when $x = 4$ km and $y = 3$ km.

(b) Find the rate of change, in km/hr, of the distance between the two ships when $x = 4$ km and $y = 3$ km.

(c) Let θ be the angle shown in the figure. Find the rate of change of θ, in radians per hour, when $x = 4$ km and $y = 3$ km.

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Question 4

Qualification: AP Calculus AB

Areas: Differentiation, Applications of Differentiation

Subtopics: Implicit Differentiation, Tangents To Curves, Local or Relative Minima and Maxima

Paper: Part B-Non-Calc / Series: 2004 / Difficulty: Very Hard / Question Number: 4

4. Consider the curve given by $x^2 + 4y^2 = 7 + 3xy$.

(a) Show that $\dfrac{dy}{dx} = \dfrac{3y - 2x}{8y - 3x}$.

(b) Show that there is a point P with x-coordinate 3 at which the line tangent to the curve at P is horizontal. Find the y-coordinate of P.

(c) Find the value of $\dfrac{d^2y}{dx^2}$ at the point P found in part (b). Does the curve have a local maximum, a local minimum, or neither at the point P? Justify your answer.

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Question 5

Qualification: AP Calculus AB

Areas: Differentiation, Applications of Differentiation

Subtopics: Implicit Differentiation, Tangents To Curves, Differentiation Technique – Chain Rule

Paper: Part B-Non-Calc / Series: 2005-Form-B / Difficulty: Somewhat Challenging / Question Number: 5

5. Consider the curve given by $y^2 = 2 + xy$.

(a) Show that $\dfrac{dy}{dx} = \dfrac{y}{2y - x}$.

(b) Find all points (x, y) on the curve where the line tangent to the curve has slope $\dfrac{1}{2}$.

(c) Show that there are no points (x, y) on the curve where the line tangent to the curve is horizontal.

(d) Let x and y be functions of time t that are related by the equation $y^2 = 2 + xy$. At time $t = 5$, the value of y is 3 and $\dfrac{dy}{dt} = 6$. Find the value of $\dfrac{dx}{dt}$ at time $t = 5$.

Question 6

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Differentiation

Subtopics: Rates of Change (Instantaneous), Implicit Differentiation, Modelling Situations, Global or Absolute Minima and Maxima, Related Rates

Paper: Part A-Calc / Series: 2008 / Difficulty: Medium / Question Number: 3

3. Oil is leaking from a pipeline on the surface of a lake and forms an oil slick whose volume increases at a constant rate of 2000 cubic centimeters per minute. The oil slick takes the form of a right circular cylinder with both its radius and height changing with time. (Note: The volume V of a right circular cylinder with radius r and height h is given by $V = \pi r^2 h$.)

   (a) At the instant when the radius of the oil slick is 100 centimeters and the height is 0.5 centimeter, the radius is increasing at the rate of 2.5 centimeters per minute. At this instant, what is the rate of change of the height of the oil slick with respect to time, in centimeters per minute?

   (b) A recovery device arrives on the scene and begins removing oil. The rate at which oil is removed is $R(t) = 400\sqrt{t}$ cubic centimeters per minute, where t is the time in minutes since the device began working. Oil continues to leak at the rate of 2000 cubic centimeters per minute. Find the time t when the oil slick reaches its maximum volume. Justify your answer.

   (c) By the time the recovery device began removing oil, 60,000 cubic centimeters of oil had already leaked. Write, but do not evaluate, an expression involving an integral that gives the volume of oil at the time found in part (b).

Question 7

Qualification: AP Calculus AB

Areas: Differentiation, Applications of Differentiation

Subtopics: Implicit Differentiation, Tangents To Curves, Differentiation Technique – Standard Functions, Vertical Tangents

Paper: Part B-Non-Calc / Series: 2008-Form-B / Difficulty: Somewhat Challenging / Question Number: 6

6. Consider the closed curve in the xy-plane given by

$$x^2 + 2x + y^4 + 4y = 5.$$

(a) Show that $\dfrac{dy}{dx} = \dfrac{-(x+1)}{2(y^3+1)}$.

(b) Write an equation for the line tangent to the curve at the point $(-2, 1)$.

(c) Find the coordinates of the two points on the curve where the line tangent to the curve is vertical.

(d) Is it possible for this curve to have a horizontal tangent at points where it intersects the x-axis? Explain your reasoning.

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Question 8

Qualification: AP Calculus AB

Areas: Differentiation, Applications of Integration

Subtopics: Modelling Situations, Total Amount, Implicit Differentiation, Interpreting Meaning in Applied Contexts, Fundamental Theorem of Calculus (First)

Paper: Part A-Calc / Series: 2009-Form-B / Difficulty: Easy / Question Number: 1

1. At a certain height, a tree trunk has a circular cross section. The radius $R(t)$ of that cross section grows at a rate modeled by the function

$$\frac{dR}{dt} = \frac{1}{16}\left(3 + \sin\left(t^2\right)\right) \text{ centimeters per year}$$

for $0 \le t \le 3,$ where time t is measured in years. At time $t = 0,$ the radius is 6 centimeters. The area of the cross section at time t is denoted by $A(t).$

(a) Write an expression, involving an integral, for the radius $R(t)$ for $0 \le t \le 3.$ Use your expression to find $R(3).$

(b) Find the rate at which the cross-sectional area $A(t)$ is increasing at time $t = 3$ years. Indicate units of measure.

(c) Evaluate $\int_0^3 A'(t)\, dt.$ Using appropriate units, interpret the meaning of that integral in terms of cross-sectional area.

Question 9

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Differentiation

Subtopics: Kinematics (Displacement, Velocity, and Acceleration), Rates of Change (Average), Riemann Sums – Left , Mean Value Theorem, Implicit Differentiation

Paper: Part B-Non-Calc / Series: 2011-Form-B / Difficulty: Medium / Question Number: 5

t (seconds)	0	10	40	60
$B(t)$ (meters)	100	136	9	49
$v(t)$ (meters per second)	2.0	2.3	2.5	4.6

5. Ben rides a unicycle back and forth along a straight east-west track. The twice-differentiable function B models Ben's position on the track, measured in meters from the western end of the track, at time t, measured in seconds from the start of the ride. The table above gives values for $B(t)$ and Ben's velocity, $v(t)$, measured in meters per second, at selected times t.

(a) Use the data in the table to approximate Ben's acceleration at time $t = 5$ seconds. Indicate units of measure.

(b) Using correct units, interpret the meaning of $\int_0^{60} |v(t)|\, dt$ in the context of this problem. Approximate $\int_0^{60} |v(t)|\, dt$ using a left Riemann sum with the subintervals indicated by the data in the table.

(c) For $40 \le t \le 60$, must there be a time t when Ben's velocity is 2 meters per second? Justify your answer.

(d) A light is directly above the western end of the track. Ben rides so that at time t, the distance $L(t)$ between Ben and the light satisfies $(L(t))^2 = 12^2 + (B(t))^2$. At what rate is the distance between Ben and the light changing at time $t = 40$?

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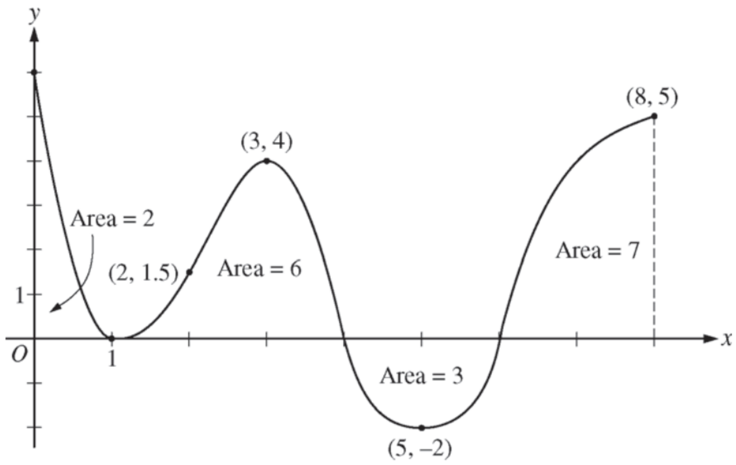
Question 10

Qualification: AP Calculus AB

Areas: Applications of Differentiation

Subtopics: Local or Relative Minima and Maxima, Global or Absolute Minima and Maxima, Concavity, Increasing/Decreasing , Implicit Differentiation, Tangents To Curves, Derivative Graphs

Paper: Part B-Non-Calc / Series: 2013 / Difficulty: Somewhat Challenging / Question Number: 4



Graph of f'

4. The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the closed interval $0 \le x \le 8$. The graph of f' has horizontal tangent lines at $x = 1$, $x = 3$, and $x = 5$. The areas of the regions between the graph of f' and the x-axis are labeled in the figure. The function f is defined for all real numbers and satisfies $f(8) = 4$.

(a) Find all values of x on the open interval $0 < x < 8$ for which the function f has a local minimum. Justify your answer.

(b) Determine the absolute minimum value of f on the closed interval $0 \le x \le 8$. Justify your answer.

(c) On what open intervals contained in $0 < x < 8$ is the graph of f both concave down and increasing? Explain your reasoning.

(d) The function g is defined by $g(x) = (f(x))^3$. If $f(3) = -\frac{5}{2}$, find the slope of the line tangent to the graph of g at $x = 3$.

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Question 11

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Integration

Subtopics: Rates of Change (Average), Intermediate Value Theorem, Riemann Sums – Trapezoidal Rule, Rates of Change (Instantaneous), Modelling Situations, Implicit Differentiation

Paper: Part B-Non-Calc / Series: 2014 / Difficulty: Somewhat Challenging / Question Number: 4

t (minutes)	0	2	5	8	12
$v_A(t)$ (meters / minute)	0	100	40	–120	–150

4. Train A runs back and forth on an east-west section of railroad track. Train A's velocity, measured in meters per minute, is given by a differentiable function $v_A(t)$, where time t is measured in minutes. Selected values for $v_A(t)$ are given in the table above.

(a) Find the average acceleration of train A over the interval $2 \le t \le 8$.

(b) Do the data in the table support the conclusion that train A's velocity is -100 meters per minute at some time t with $5 < t < 8$? Give a reason for your answer.

(c) At time $t = 2$, train A's position is 300 meters east of the Origin Station, and the train is moving to the east. Write an expression involving an integral that gives the position of train A, in meters from the Origin Station, at time $t = 12$. Use a trapezoidal sum with three subintervals indicated by the table to approximate the position of the train at time $t = 12$.

(d) A second train, train B, travels north from the Origin Station. At time t the velocity of train B is given by $v_B(t) = -5t^2 + 60t + 25$, and at time $t = 2$ the train is 400 meters north of the station. Find the rate, in meters per minute, at which the distance between train A and train B is changing at time $t = 2$.

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Question 12

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Differentiation

Subtopics: Implicit Differentiation, Differentiation Technique – Trigonometry, Differentiation Technique – Product Rule, Tangents To Curves, Local or Relative Minima and Maxima, Differentiation Technique - Quotient Rule

Paper: Part B-Non-Calc / Series: 2021 / Difficulty: Hard / Question Number: 5

5. Consider the function $y = f(x)$ whose curve is given by the equation $2y^2 - 6 = y \sin x$ for $y > 0$.

(a) Show that $\dfrac{dy}{dx} = \dfrac{y \cos x}{4y - \sin x}$.

(b) Write an equation for the line tangent to the curve at the point $(0, \sqrt{3})$.

(c) For $0 \leq x \leq \pi$ and $y > 0$, find the coordinates of the point where the line tangent to the curve is horizontal.

(d) Determine whether f has a relative minimum, a relative maximum, or neither at the point found in part (c). Justify your answer.

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Question 13

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Differentiation, Integration

Subtopics: Rates of Change (Average), Intermediate Value Theorem, Riemann Sums – Right, Implicit Differentiation, Rates of Change (Instantaneous), Differentiation Technique – Chain Rule, Related Rates

Paper: Part B-Non-Calc / Series: 2022 / Difficulty: Medium / Question Number: 4

t (days)	0	3	7	10	12
$r'(t)$ (centimeters per day)	-6.1	-5.0	-4.4	-3.8	-3.5

4. An ice sculpture melts in such a way that it can be modeled as a cone that maintains a conical shape as it decreases in size. The radius of the base of the cone is given by a twice-differentiable function r, where $r(t)$ is measured in centimeters and t is measured in days. The table above gives selected values of $r'(t)$, the rate of change of the radius, over the time interval $0 \le t \le 12$.

(a) Approximate $r''(8.5)$ using the average rate of change of r' over the interval $7 \le t \le 10$. Show the computations that lead to your answer, and indicate units of measure.

(b) Is there a time t, $0 \le t \le 3$, for which $r'(t) = -6$? Justify your answer.

(c) Use a right Riemann sum with the four subintervals indicated in the table to approximate the value of $\int_0^{12} r'(t)\, dt$.

(d) The height of the cone decreases at a rate of 2 centimeters per day. At time $t = 3$ days, the radius is 100 centimeters and the height is 50 centimeters. Find the rate of change of the volume of the cone with respect to time, in cubic centimeters per day, at time $t = 3$ days. (The volume V of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.)

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Question 14

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Differentiation

Subtopics: Implicit Differentiation, Tangents To Curves, Vertical Tangents, Modelling Situations

Paper: Part B-Non-Calc / Series: 2023 / Difficulty: Hard / Question Number: 6

6. Consider the curve given by the equation $6xy = 2 + y^3$.

(a) Show that $\frac{dy}{dx} = \frac{2y}{y^2 - 2x}$.

(b) Find the coordinates of a point on the curve at which the line tangent to the curve is horizontal, or explain why no such point exists.

(c) Find the coordinates of a point on the curve at which the line tangent to the curve is vertical, or explain why no such point exists.

(d) A particle is moving along the curve. At the instant when the particle is at the point $\left(\frac{1}{2}, -2\right)$, its horizontal position is increasing at a rate of $\frac{dx}{dt} = \frac{2}{3}$ unit per second. What is the value of $\frac{dy}{dt}$, the rate of change of the particle's vertical position, at that instant?

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Question 15

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Differentiation

Subtopics: Tangents To Curves, Vertical Tangents, Implicit Differentiation, Rates of Change (Instantaneous)

Paper: Part B-Non-Calc / Series: 2024 / Difficulty: Somewhat Challenging / Question Number: 5

5. Consider the curve defined by the equation $x^2 + 3y + 2y^2 = 48$. It can be shown that $\frac{dy}{dx} = \frac{-2x}{3 + 4y}$.

(a) There is a point on the curve near $(2, 4)$ with x-coordinate 3. Use the line tangent to the curve at $(2, 4)$ to approximate the y-coordinate of this point.

(b) Is the horizontal line $y = 1$ tangent to the curve? Give a reason for your answer.

(c) The curve intersects the positive x-axis at the point $(\sqrt{48}, 0)$. Is the line tangent to the curve at this point vertical? Give a reason for your answer.

(d) For time $t \geq 0$, a particle is moving along another curve defined by the equation $y^3 + 2xy = 24$. At the instant the particle is at the point $(4, 2)$, the y-coordinate of the particle's position is decreasing at a rate of 2 units per second. At that instant, what is the rate of change of the x-coordinate of the particle's position with respect to time?

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